

The scaling infrared DSE solution as a critical end-point for the family of decoupling ones

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Abstract.

Both regular (the zero-momentum ghost dressing function not diverging), also named decoupling, and critical (diverging), also named scaling, Yang-Mills propagators solutions can be obtained by analyzing the low-momentum behaviour of the ghost propagator Dyson-Schwinger equation (DSE) in Landau gauge. The asymptotic expression obtained for the regular or decoupling ghost dressing function up to the order $\mathcal{O}(q^2)$ fits pretty well the low-momentum ghost propagator obtained through the numerical integration of the coupled gluon and ghost DSE in the PT-BFM scheme. Furthermore, when the size of the coupling renormalized at some scale approaches some critical value, the PT-BFM results seems to tend to the scaling solution as a limiting case. This critical value of the coupling is compared with the lattice estimate for the Yang-Mills QCD coupling and the latter is shown to lie much above the former.

Keywords: Dyson-Schwinger equations, Infrared QCD

PACS: 12.35.Aw, 12.38.Lg, 12.38.Gc

INTRODUCTION

The low-momentum behaviour of the Yang-Mills propagators derived either from the tower of Dyson-Schwinger equations (DSE) or from Lattice simulations in Landau gauge has been a very interesting and hot topic for the last few years. It seems by now well established that, if we assume in the vanishing momentum limit a ghost dressing function behaving as $F(q^2) \sim (q^2)^{\alpha_F}$ and a gluon propagator as $\Delta(q^2) \sim (q^2)^{\alpha_G-1}$ (or, by following a notation commonly used, a gluon dressing function as $G(q^2) = q^2 \Delta(q^2) \sim (q^2)^{\alpha_G}$), two classes of solutions may emerge (see, for instance, the discussion of refs. [1, 2]) from the DSE: (i) those, dubbed “*decoupling*”, where $\alpha_F = 0$ and the suppression of the ghost contribution to the gluon propagator DSE results in a massive gluon propagator (see [3, 4] and references therein); and (ii) those, dubbed “*scaling*”, where $\alpha_F \neq 0$ and the low-momentum behaviour of both gluon and ghost propagators are related by the coupled system of DSE through the condition $2\alpha_F + \alpha_G = 0$ implying that $F^2(q^2)G(q^2)$ goes to a non-vanishing constant when $q^2 \rightarrow 0$ (see [5, 6] and references therein).

Lattice QCD results appear to support only the massive gluon ($\alpha_G = 1$) or scaling solutions (see [11] and references therein), and also pinching technique results (see, for instance, [12, 18] and references therein), refined Gribov-Zwanziger¹ formalism (see [13]) or other

approaches like the infrared mapping of $\lambda\phi^4$ and Yang-Mills theories in ref. [14] or the massive extension of the Fadeev-Popov action in ref. [15] appear to point to.

In the present contribution, we will briefly review the work of ref. [17], which extended the previous studies of refs. [1, 2, 16], by the analysis of the results obtained by solving the coupled system of Landau gauge ghost and gluon propagators DSE within the framework of the pinching technique in the background field method [18] (PT-BFM)

THE TWO KINDS OF SOLUTIONS OF THE GHOST PROPAGATOR DYSON-SCHWINGER EQUATION

As was explained in detail in refs. [2, 16, 17], the low-momentum behavior for the Landau gauge ghost dressing function can be inferred from the analysis of the Dyson-Schwinger equation for the ghost propagator (GPDSE). That analysis is performed on a very general ground: one applies the MOM renormalization prescription, $F_R(\mu^2) = \mu^2 \Delta_R(\mu^2) = 1$, where μ^2 is the subtraction point, chooses for the ghost-gluon vertex,

$$\begin{aligned} \tilde{\Gamma}_v^{abc}(-q, k; q-k) &= ig_0 f^{abc} (q_v H_1(q, k) \\ &+ (q-k)_v H_2(q, k)) \end{aligned} \quad (1)$$

to apply this MOM prescription in Taylor kinematics (*i.e.* with a vanishing incoming ghost momentum) and assumes the non-renormalizable bare ghost-gluon form factor, $H_1(q, k) = H_1$, to be constant in the low-momentum regime for the incoming ghost. Then, the low

¹ In addition, K-I. Kondo triggered very recently an interesting discussion about the Gribov horizon condition and its implications on the Landau-gauge Yang-Mills infrared solutions [7, 8, 9, 10].

momentum-behaviour of the ghost dressing function is supposed to be well described by

$$F_R(q^2) = A(\mu^2) \left(\frac{q^2}{M^2} \right)^{\alpha_F} \left(1 + \dots \right), \quad (2)$$

and that of the gluon propagator by

$$\Delta_R(q^2) = \frac{B(\mu^2)}{q^2 + M^2} \simeq \frac{B(\mu^2)}{M^2} \left(1 - \frac{q^2}{M^2} + \dots \right) \quad (3)$$

and this, after solving asymptotically the GPDSE, finally left us with:

$$F_R(q^2) \simeq \left(\frac{10\pi^2}{N_C H_1 g_R(\mu^2) B(\mu^2)} \right)^{1/2} \left(\frac{M^2}{q^2} \right)^{1/2}, \quad (4)$$

if $\alpha_F \neq 0$; and

$$F_R(q^2) \simeq F_R(0) \left(1 + \frac{N_C H_1}{16\pi} \bar{\alpha}_T(0) \frac{q^2}{M^2} \left[\ln \frac{q^2}{M^2} - \frac{11}{6} \right] + \mathcal{O} \left(\frac{q^4}{M^4} \right) \right) \quad (5)$$

if $\alpha_F = 0$, where

$$\bar{\alpha}_T(0) = M^2 \frac{g_R^2(\mu^2)}{4\pi} F_R^2(0) \Delta_R(0). \quad (6)$$

It should be understood that the subtraction momentum for all the renormalization quantities is μ^2 . The case $\alpha_F \neq 0$ leads to the so-called scaling solution, where the low-momentum behavior of the massive gluon propagator forces the ghost dressing function to diverge at low-momentum through the scaling condition: $2\alpha_F + \alpha_G = 0$ ($\alpha_G = 1$ is the power exponent when dealing with a massive gluon propagator). As this scaling condition is verified, the perturbative strong coupling defined in this Taylor scheme [19], $\alpha_T = g_T^2/(4\pi)$, has to reach a constant at zero-momentum,

$$\begin{aligned} \alpha_T(0) &= \frac{g^2(\mu^2)}{4\pi} \lim_{q^2 \rightarrow 0} q^2 \Delta(q^2) F^2(q^2) \\ &= \frac{5\pi}{2N_C H_1}, \end{aligned} \quad (7)$$

as can be obtained from Eqs.(3,5). The case $\alpha_F = 0$ corresponds to the so-called decoupling solution, where the zero-momentum ghost dressing function reaches a non-zero finite value and eq. (5) provides us with the first asymptotic corrections to this leading constant. This sub-leading correction is controlled by the zero-momentum value of the coupling defined in eq. (6), which is an extension of the non-perturbative effective charge definition from the gluon propagator [20] to the Taylor ghost-gluon coupling [21].

TABLE 1. Gluon masses and the zero-momentum non-perturbative effective charges, taken from ref. [17] and obtained as discussed in the text.

$\alpha(\mu)$	$\bar{\alpha}_T(0)$	M (GeV) [gluon]
0.15	0.24	0.37
0.16	0.30	0.39
0.17	0.41	0.43

NUMERICAL RESULTS FROM COUPLED PT-BFM DSE'S

In ref. [17], the solutions of the coupled DSE system in the PT-BFM scheme (with $H_1 = 1$ for the ghost-gluon vertex), numerically integrated for many values of the coupling at the renormalization point μ^2 as a boundary condition, were studied in the light of the analytical results above presented. Here we will shortly discuss the results of this work.

The “regular” or “decoupling” solutions

The numerical results of the PT-BFM coupled DSEs were shown in ref. [17] to behave asymptotically as eq. (4) predicts for the decoupling DSE solutions. Indeed, as the gluon propagator solutions in the PT-BFM scheme result to behave as massive ones, the eqs. (3,5) must account for the low-momentum behaviour of both gluon propagator and ghost dressing function with $H_1 = 1$ and $\bar{\alpha}_T(0)$ given by eq. (6), with $\alpha_T(\mu^2) = g_R^2(\mu^2)/(4\pi)$ being fixed, as a boundary condition for the numerical integration of the coupled DSE for each particular solution of the family (see tab. 1). Furthermore, the zero-momentum values of the ghost dressing function, $F_R(0)$ and of the gluon propagator, $\Delta_R(0)$, can be taken from the numerical solutions of the DSE (for any value of the $\alpha(\mu = 10\text{GeV})$). These altogether with the gluon masses obtained by the fit of eq. (3) to the numerical DSE gluon propagator solutions (see the left plot in fig. 1, for $\alpha(\mu) = 0.16$, and the results for $\alpha(\mu) = 0.15, 0.16, 0.17$ in tab. 1, taken from ref. [17]), provide us with all the ingredients to evaluate, with no unknown parameter, eq. (5).

Indeed, the expression given by eq. (5) can be successfully applied to describe the solutions all over the range of coupling values, $\alpha(\mu)$, at $\mu = 10$ GeV (provided that they are not very close of the critical coupling that will be defined in the next subsection). This can be seen, for instance, for $\alpha = 0.16, 0.18$, in the right plots of fig. 1 and it was also shown for $\alpha = 0.15$ in ref. [17].

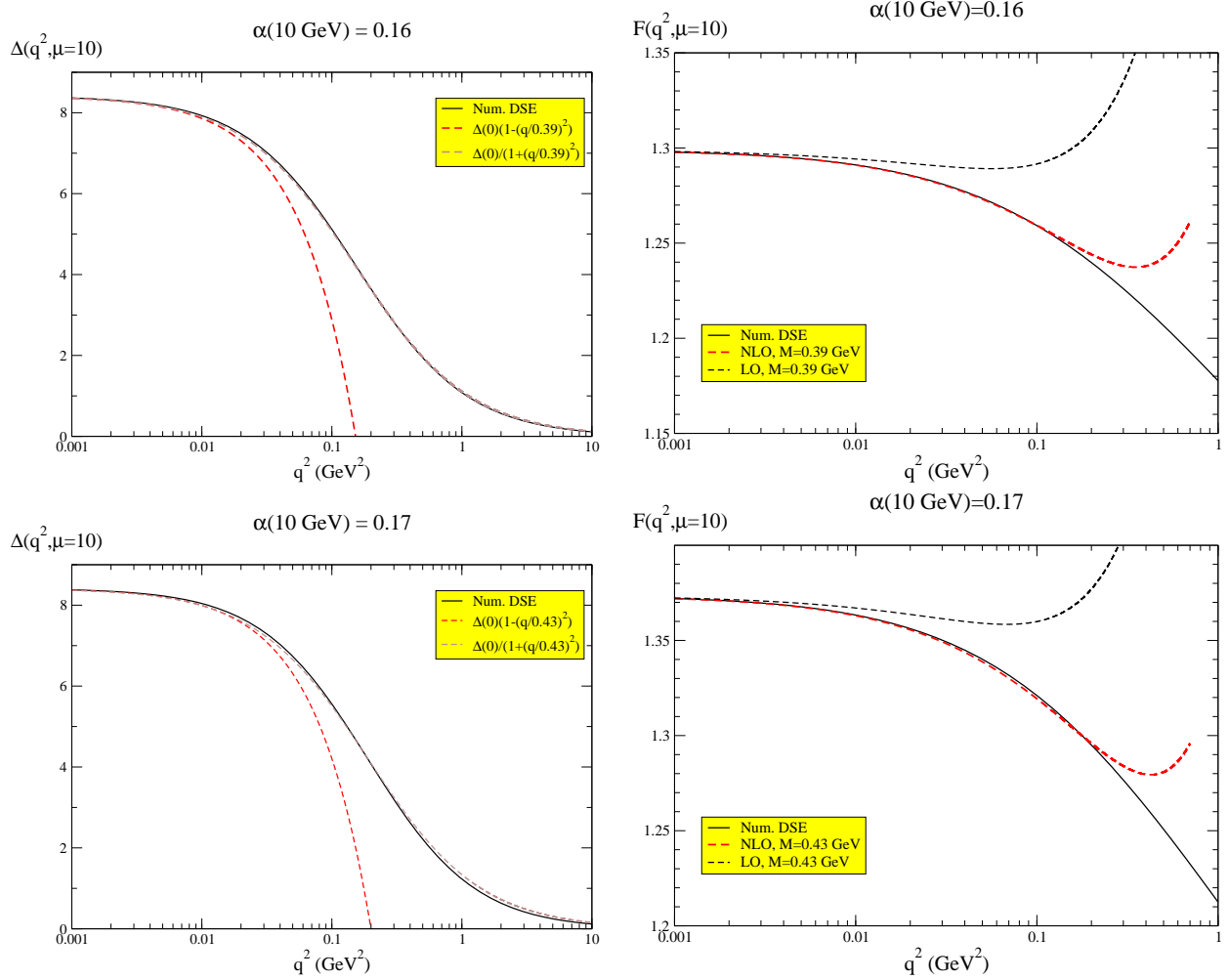


FIGURE 1. Gluon propagator (left) and ghost dressing function (right) after the numerical integration of the coupled DSE system for $\alpha(\mu = 10\text{GeV}) = 0.16, 0.17$ taken from [17]. The curves for the best fits to gluon propagator and ghost propagator data explained in the text appear as red dotted lines. the same for the black dotted line in the lefthand plot but retaining only the logarithmic leading term for the asymptotic ghost dressing function by dropping the $-11/6$ away.

The “critical” limit

There also appeared to be a *critical* value of the coupling, $\alpha_{\text{crit}} = \alpha(\mu^2) \simeq 0.182$ with $\mu = 10\text{ GeV}$, above which the coupled DSE system does not converge any longer to a solution. As a matter of fact, we know from refs.[2, 17] that the scaling solution implies for the coupling

$$\alpha_{\text{crit}} = \frac{g_R^2(\mu^2)}{4\pi} \simeq \frac{5\pi}{2N_C A^2(\mu^2) B(\mu^2)}, \quad (8)$$

where $B(\mu^2)$ and $A(\mu^2)$ defined by Eqs. (3,4). This is also shown in ref. [1], where only the ghost propagator DSE with the kernel for the gluon loop integral is obtained from gluon propagator lattice data. In the analy-

sis of ref. [1], a ghost dressing function solution diverging at vanishing momentum appears to exist and verifies eqs. (4,8), while regular or decoupling solutions exist for any $\alpha < \alpha_{\text{crit}}$. In ref. [17], a more complete analysis is performed: first by studying the solutions for many different values of the coupling, $\alpha = \alpha(\mu^2)$, of a coupled DSE system; and then by showing that the ghost dressing function at vanishing momentum, $F(0, \mu^2)$, is described by the following power behaviour,

$$F(0) \sim (\alpha_{\text{crit}} - \alpha(\mu^2))^{-\kappa(\mu^2)}, \quad (9)$$

where $\kappa(\mu^2)$ is a critical exponent (depending presumably on the renormalization point, μ^2), supposed to be positive and to govern the transition from decoupling ($\alpha < \alpha_{\text{crit}}$) to the scaling ($\alpha = \alpha_{\text{crit}}$) solutions;

and where we let α_{crit} be a free parameter to be fitted by requiring the best linear correlation for $\log[F(0)]$ in terms of $\log[\alpha_{\text{crit}} - \alpha]$. In doing so, the best correlation coefficient is 0.9997 for $\alpha_{\text{crit}} = 0.1822$, which is pretty close to the critical value of the coupling above which the coupled DSE system does not converge any more, and $\kappa(\mu^2) = 0.0854(6)$. This can be seen in fig. 2, where the log-log plot of $F_R(0)$ in terms of $\alpha_{\text{crit}} - \alpha$ is shown and the linear behaviour with negative slope corresponding to the best correlation coefficient strikingly indicates a zero-momentum ghost propagator diverging as $\alpha \rightarrow \alpha_{\text{crit}}$. Nevertheless, no critical or scaling solution appears for the coupled DSE system in the PT-BFM, although the decoupling solutions obtained for any $\alpha < \alpha_{\text{crit}} = 0.1822$ seem to approach the behaviour of a scaling one when $\alpha \rightarrow \alpha_{\text{crit}}$. This is again well understood in ref. [17], where the gluon propagators obtained from the coupled DSE system in PT-BFM were also found to obey the same critical behaviour pattern as the ghost propagator, when approaching the critical value of the coupling. Indeed, when approaching the critical value of the coupling, the gluon propagators obtained from the coupled DSE system in PT-BFM must be also thought to obey the same critical behaviour pattern as the ghost propagator. In the PT-BFM, the value at zero-momentum being fixed by construction [4], one should expect that, instead of decreasing, the gluon propagator obtained for couplings near to the critical value increases for low momenta: the more one approaches the critical coupling the more it has to increase. This is indeed the case, as can be seen in fig. 2(b). This also implies that, near the critical value, the low momentum propagator does not obey eq. (3) and that consequently eq. (5) does not work any longer to describe the low momentum ghost propagator.

The critical value and the QCD coupling in the “real world”

Finally, one can pay attention to the critical value of the coupling, $\alpha_{\text{crit}} = 0.1822$, and try to make a comparison with the physical strong coupling values in order get some idea of whether the current data can exclude or not this critical behaviour. Although the experimental PDG *world average* of the strong coupling in the $\overline{\text{MS}}$ scheme, $\alpha_{\overline{\text{MS}}}(M_Z) = 0.1184(7)$ [22], can be propagated from the Z^0 boson mass down to $\mu = 10$ GeV to give $\alpha_{\overline{\text{MS}}}(10 \text{ GeV}) = 0.179(2)$, that incidentally lies on the right ballpark of the above critical value, such a comparison is meaningless because our coupling corresponds to one in MOM Taylor-scheme for zero number

of flavours. One can use instead the available perturbative four-loop formula describing the running of the coupling in Taylor-scheme to estimate Λ_{QCD} in this particular scheme, then perform the conversion to $\overline{\text{MS}}$ (see for instance eqs.(22,23) of the first reference in [19]) and thus obtain the value quoted in tab. 2. Of course, it would be again meaningless to compare this last value with the one for $\Lambda_{\overline{\text{MS}}}$ that can be obtained from the PDG value for $\alpha_{\overline{\text{MS}}}(M_Z)$, also quoted in tab. 2, but we can refer the comparison to the lattice Yang-Mills determinations of the same parameter, as for instance the two of them included in tab. 2. It should be noted that the procedures for the lattice determination of $\Lambda_{\overline{\text{MS}}}$ mainly work in the UV domain, where IR sources of uncertainties as the Gribov ambiguity or volume effects are indeed negligible². Thus, the lattice estimates of $\Lambda_{\overline{\text{MS}}}$ appear to lie clearly below this critical limit for the PT-BFM DSE in pure Yang-Mills. This last results appears to indicate that the critical solution is not the one chosen by the zero-flavour world. However, as no quark flavour loops effect have been incorporated in our DSE analysis, we cannot yet neither compare with the physical strong coupling nor conclude whether the critical limit can be allowed in the “real world”.

TABLE 2. The critical value of $\Lambda_{\overline{\text{MS}}}$ in pure Yang-Mills inferred from $\alpha_{\text{crit}} = 0.1822$ (first column), lattice estimates for Yang-Mills $\Lambda_{\overline{\text{MS}}}$ taken from literature (second and third columns) and the one obtained from the PDG value of $\alpha_{\overline{\text{MS}}}(M_Z)$ by applying a four-loop perturbative formula for the running of $\alpha_{\overline{\text{MS}}}$ with $N_f = 5$.

$\Lambda_{\overline{\text{MS}},\text{crit}}^{N_f=0}$	$\Lambda_{\overline{\text{MS}}}^{N_f=0}$ [24]	$\Lambda_{\overline{\text{MS}}}^{N_f=0}$ [19]	$\Lambda_{\overline{\text{MS}}}^{N_f=5}$ [22]
434 MeV	238(19) MeV	244(8) MeV	213(9) MeV

CONCLUSIONS

The ghost propagator DSE, with the only assumption of taking $H_1(q, k)$ from the ghost-gluon vertex in eq. (1) to be constant in the infrared domain of q , can be exploited to look into the low-momentum behaviour of the ghost propagator. The two classes of solutions named “decoupling” and “scaling” can be indentified and shown to depend on whether the ghost dressing function achieves a finite non-zero constant ($\alpha_F = 0$) at vanishing momentum or not ($\alpha_F \neq 0$). The solutions appear to be dialed by the size of the coupling at the renormalization momentum which plays the role of a boundary condition for the

² As a matter of fact, there are unquenched lattice determinations with $N_f = 5$ staggered fermions for the strong coupling [23] which are pretty consistent with the PDG value. This can be taken as a good indication in favour of the robustness of the lattice determinations of $\Lambda_{\overline{\text{MS}}}$.

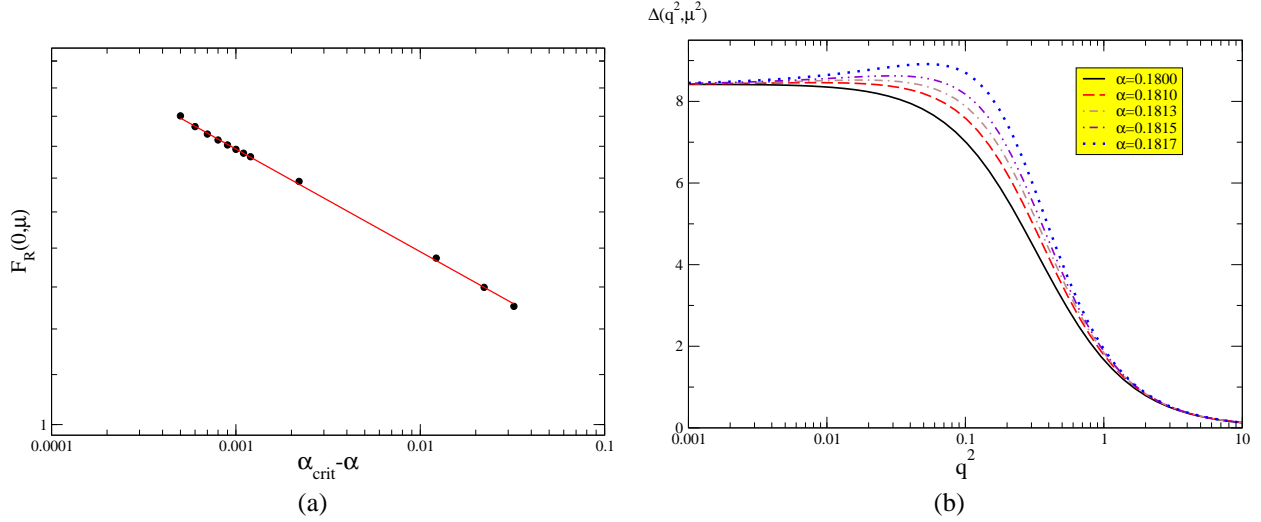


FIGURE 2. (a) Log-log plot of the zero-momentum values of the ghost dressing function, obtained by the numerical integration of the coupled DSE system in the PT-BFM scheme, in terms of $\alpha_{\text{crit}} - \alpha$. $\alpha = \alpha(\mu = 10\text{GeV})$, the value of the coupling at the renormalization momentum, is an initial condition for the integration; while α_{crit} is fixed to be 0.1822, as explained in the text, by requiring the best linear correlation. (b) Gluon propagator solutions in terms of q^2 for the same coupled DSE system for different values of $\alpha(\mu = 10\text{GeV})$, all very close to the critical value, ranging from 0.18 to 0.1817.

DSE integration. The low-momentum behaviour of the decoupling solutions results to be regulated by the gluon propagator mass and by a regularization-independent dimensionless quantity that appears to be the effective charge defined from the Taylor-scheme ghost-gluon vertex at zero momentum.

In this note, we have shortly discussed the results of ref. [17] where the solutions of coupled ghost and gluon propagator DSE in the PT-BFM scheme were studied and demonstrated that the asymptotic decoupling formula ($\alpha_F = 0$) successfully describes the low-momentum ghost propagator. The model applied for the massive gluon propagator is also verified to give properly account of the gluon solution, at least for momenta below 1 GeV (and for a coupling not very close to the critical point). Although we argued that a massive gluon propagator implies that the ghost dressing function takes a non-zero finite value at vanishing momentum, we also show that the zero-momentum ghost dressing function tends to diverge when the value of the coupling dialing the solutions approaches some critical value. Such a divergent behaviour at the critical coupling seems to be the expected one for a scaling solution (where, if the gluon is massive, $\alpha_F = -1/2$). If we consider the zero-momentum value of the ghost dressing function as some sort of “*order parameter*” indicating whether the ghost propagator low-momentum behaviour is suppressed ($\alpha_F = 0$ and finite ghost dressing function) or it is enhanced ($\alpha_F < 0$ and divergent ghost dressing function), the strength of the coupling computed at some renormalization point seems to control some sort of tran-

sition from the *suppressed* to the *enhanced* phases for the ghost propagator DSE solutions in the PT-BFM scheme. The last only takes place as some critical value of the coupling is reached. Nevertheless, it can be proven that, as far as the gluon is massive, the scaling behaviour for the Yang-Mills propagators appear not to be a solution but an unattainable limiting case for the PT-BFM DSE solutions.

Finally, the critical value for the coupling is shown to lie clearly much above the estimate of the QCD coupling in pure Yang-Mills computed from lattice QCD. This of course agrees with the fact that the current large-volume quenched lattice results for ghost and gluon Green functions clearly behave as expected for the decoupling solutions.

ACKNOWLEDGMENTS

The author is particularly indebted to Ph. Boucaud, J.P. Leroy, A. Le Yaouanc, J. Micheli and O. Pène for very fruitful discussions at the initial stages of the work and to J. Papavassiliou and A.C. Aguilar also for very valuable discussions and comments, and specially for providing me with some unpublished results which were exploited in this paper. J. R-Q also acknowledges the Spanish MICINN for the support by the research project FPA2009-10773 and “Junta de Andalucía” by P07FQM02962.

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